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Two-Mode Harmonic Oscillator Model of Reissner-Nordström Black Hole

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Abstract In this paper, we will assume that the Reissner-Nordström black hole can be dynamically expressed, at the classical level, in terms of a reduced phase space consisting of the physical observables and their canonical conjugates. More specifically, we propose that the phase space can be described by the black hole's mass and charge along with their respective conjugates. In this four-dimensional phase space we perform a couple of canonical transformations to obtain a two-mode harmonic oscillator model of Reissner-Nordström black hole. By adopting this model, the quantum spectrum and wave function of horizon area are presented.

Keywords Reissner-Nordström black hole · Reduced phase space · Canonical transformations · Quantum spectrum

1 Introduction

There has been significant interest, over the last decades, in the subject known as black-hole thermodynamics. Since Bekenstein's [1] and Hawking's [2] seminal work, the thermodynamics and statistic mechanics of black hole have been one of the important research fields. Especially in recent years, researches has already obtained some interesting results [3–7]. Bekenstein firstly advocated that the area spectrum of the black hole is discrete and uniformly spaced. This propose is based on the observation that the horizon area of black hole is an adiabatic invariant [9]. According to Erenfest principle of the old quantum theory, an adiabatic invariant corresponds to a quantum number. In 1974, Bekenstein have already argued that the possible eigenvalues of the black hole horizon area are of the form [8]

$$A_n = \gamma n l_{pl}^2, \tag{1}$$

where n is an integer, γ is a numerical factor of the order unity and l_{pl} is the Planck length. In order to approve the Bekenstein's hypothesis, many authors established different quantum

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models of black hole [10–16], and obtained the quantum area spectrums. Louko and Mäkelä have derived the quantum area spectrum of Schwarzschild black hole [11]

$$A_n \sim 32\pi k l_{pl}^2 + \text{Const} + O(1) \quad n = 0, 1, 2, \dots \tag{2}$$

where k is an integer, and when k is large, $O(1)$ tends to zero. Mäkelä and Repo have constructed a quantum dynamical model of the Rerssner-Nordström black hole through calculating its Hamiltonian. Taking the inner and outer horizons into account, they have also given out the quantum area spectrum [18]

$$A_n^{tot} \approx 32\pi n l_{pl}^2 + 2A_{ext} \quad n = 0, 1, 2, \dots \tag{3}$$

where $A_{ext} = 4\pi a^2$ is the extremal value of the horizon area. In [15–17], the authors employed the period boundary condition and the method of introducing compulsorily the conjugate variables to obtain the quantum horizon area spectrums of the Kerr and the Kerr-Newman black holes, respectively,

$$A_{n,m} = 8\pi\hbar\left(n + m + \frac{1}{2}\right) \quad n, m = 0, 1, 2, \dots, \tag{4}$$

$$A_{n,p_1,p_2} = 8\pi\hbar\left(n + \frac{p_1}{2} + p_2 + \frac{1}{2}\right) \quad n, p_1, p_2 = 0, 1, 2, \dots. \tag{5}$$

In [19], they also have gotten the quantum area spectrum of de Sitter space-time.

$$A_c = 8\pi\left(n + \frac{1}{2}\right) \quad n = 0, 1, 2, \dots. \tag{6}$$

All the results above have confirmed that the horizon area of black hole is quantized. In these models, especially, a model called "reduced phase space quantization" [20] looks simple and elegant. In this model, some coordinate invariants, such as black hole's mass M and charge Q, are treated as the dynamical variables and their conjugates are treated as generalized momenta.

Here we attempt to consider the mass and the charge of Reissner-Nordström black hole as dynamical variables, and their conjugates as generalized momenta. Through these four variables, a four-dimensional phase space is constituted. In addition, we emphasize that one assumption goes into our quantization procedure: the conjugate to the mass is periodic in accordance with Euclidean considerations [17, 21]. Based on this simple assumption and canonical coordinate transformation, we can derive the quantum area spectrum and the entropy of Reissner-Nordström black hole. By introducing the Fock representation, we then show the holographic quantum information of Reissner-Nordström black hole.

It should be noted that, in a recent study of interest, Makelo *et al.* have similarly considered the area spectrum of the black hole. The approach of these authors is based on formulating a Schrödinger-like equation for the black hole observables and quantizing this equation by way of WKB techniques. In [22], Gout *et al.* show that the area spectrum is uniformly spaced by implementing certain selection rules. In this paper, we focused on a couple of canonical coordinate transformations, and also proved that the area of the Reissner-Nordström black hole is quantized.

This paper is organized as follows. In the second section, a two-mode Bose harmonic oscillator model of Reissner-Nordström black hole is obtained by suitable canonical transformations. By adopting this model, the quantum area spectrum and wave function of horizon area are presented in the third and fourth section. At last we will conclude our paper.

2 A Two-Mode Harmonic Oscillator Model of Reissner-Nordström Black Hole

It is well known that the space-time metric of Reissner-Nordström black hole is

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (7)$$

Following reference [17, 19], the coordinate invariants can be viewed as the dynamical variables, and ensure that the Hamiltonian is independent of the conjugate momenta. Here it is evident that there are only two observational physical quantities, the mass M and the charge Q. Employing the method of reduced phase space quantization, these two dynamical variables and their conjugate momenta constitute a four-dimensional phase space (M, Q, p_M, p_Q). The expression of the simplified action quantity [17, 18] must be

$$I = \int dt[p_M\dot{M} + p_Q\dot{Q} - H(M, Q)]. \quad (8)$$

The differential form of Bekenstein-Smarr formula in Reissner-Nordström black hole is

$$\delta M = \frac{\kappa}{8\pi}\delta A + V_0\delta Q. \quad (9)$$

According to Euclidean quantum gravity theory, the conjugate momentum of M plays the role of "time" and should have a period and the period is the inverse Hawking temperature, that is, $p_M \sim p_M + \frac{2\pi}{\kappa}$. So we now in a situation familiar in classical mechanics, where there exists a periodic, angular variable. Akin to the action-angle formulation of the harmonic oscillator, we "unwrap" our gravitational phase space by transforming to a set of unrestricted variables. Consider the following transformation $(M, Q, p_M, p_Q) \to (X, Y, \pi_X, \pi_Y)$. We do canonical coordinate transformations and set:

$$X = \sqrt{B}\cos(\kappa p_M), \quad (10)$$

$$\pi_X = \sqrt{B}\sin(\kappa p_M), \quad (11)$$

$$Y = \sqrt{2}Q\cos(\gamma p_Q), \quad (12)$$

$$\pi_Y = \sqrt{2}Q\sin(\gamma p_Q), \quad (13)$$

where $B = B(M, Q)$ and $\gamma = \gamma(Q)$ are undetermined functions, $\kappa = \kappa(Q)$ is the surface gravity. The transformations yield two pairs of nonperiod variables in a way that encorporate directly the correct periodicity of p_M and p_Q.

A straightforward calculation reveals that,

$$\begin{aligned}\pi_X\delta X = {}& \frac{1}{2}\sin(\kappa p_M)\cos(\kappa p_M)\left(\frac{\partial B}{\partial M}\delta M + \frac{\partial B}{\partial Q}\delta Q\right)\\ &- Bp_M\sin^2(\kappa p_M)\left(\frac{\partial\kappa}{\partial M}\delta M + \frac{\partial\kappa}{\partial Q}\delta Q\right) - B\kappa\sin^2(\kappa p_M)\delta p_M, \quad (14)\end{aligned}$$

$$\begin{aligned}\pi_Y\delta Y = {}& 2Q\sin(\gamma p_Q)\cos(\gamma p_Q)\delta Q - 2Q^2 p_Q\sin^2(\gamma p_Q)\frac{\partial\gamma}{\partial Q}\delta Q\\ &- 2Q^2\gamma\sin^2(\gamma p_Q)\delta Q. \quad (15)\end{aligned}$$

Using the canonical transformation

$$p_M\delta M + p_Q\delta Q - \pi_X\delta X - \pi_Y\delta Y = h_{p_M}\delta p_M + h_{p_Q}\delta p_Q + h_M\delta M + h_Q\delta Q, \tag{16}$$

where

$$h_{p_M} = B\kappa\sin^2(\kappa p_M), \tag{17}$$

$$h_{p_Q} = 2Q^2\gamma\sin^2(\gamma p_Q), \tag{18}$$

$$h_M = p_M - \frac{1}{2}\frac{\partial B}{\partial M}\sin(\kappa p_M)\cos(\kappa p_M) + Bp_M\frac{\partial\kappa}{\partial M}\sin^2(\kappa p_M), \tag{19}$$

$$h_Q = p_Q - \frac{1}{2}\frac{\partial B}{\partial Q}\sin(\kappa p_M)\cos(\kappa p_M) + Bp_M\frac{\partial\kappa}{\partial Q}\sin^2(\kappa p_M) -2Q\sin(\gamma p_Q)\cos(\gamma p_Q) + 2Q^2 p_Q\sin^2(\gamma p_Q)\frac{\partial\gamma}{\partial Q} \tag{20}$$

and according to [16], if the above transformation $(M, Q, p_M, p_Q) \to (X, Y, \pi_X, \pi_Y)$ is canonical transformations, there must be $\frac{\partial h_{p_M}}{\partial M} = \frac{\partial h_M}{\partial p_M}, \frac{\partial h_{p_Q}}{\partial Q} = \frac{\partial h_Q}{\partial p_Q}$. Then we can obtain

$$1 - \frac{1}{2}\kappa\frac{\partial B}{\partial M} = 0, \tag{21}$$

$$\gamma = \frac{1}{2Q}. \tag{22}$$

Comparing (21) with (9), we can determine the relationship of the function $B(M, Q)$ and the horizon area A by

$$\frac{\partial B(M,Q)}{\partial M} = \frac{1}{4\pi}\frac{\partial A}{\partial M}. \tag{23}$$

From (23) above, the expression of the function $B(M, Q)$ is

$$B(M,Q) = \frac{A(M,Q)}{4\pi} + F(Q) \tag{24}$$

where the $F(Q)$ is an essential arbitrary function. In spite of this freedom in choosing F, there is only one particular form that will be useful for quantization of the area [23]. First, the function B should be bounded from below regardless of the choice of F. This follows the lower bound that exits on the area A. To be precise, for a charged black hole, The (outer) horizon area of a Reissner-Nordström black hole is bounded from below-at least classically-by its extremal value; that is

$$A = 8\pi\left(M^2 - \frac{Q^2}{2} + M\sqrt{M^2 - Q^2}\right), \tag{25}$$

$$A \geq A_{ext}(Q) = 4\pi Q^2. \tag{26}$$

In order to guarantee that (25) and (26) are well defined, the function $B = B(M, Q)$ should have non-negative values. So the most suitable choice is $F(Q) = -Q^2$, through (25) and (26), we can get

$$A = 4\pi(\pi_X^2 + X^2) + 4\pi Q^2. \tag{27}$$

Substituting (27) into (12) and (13), we obtain

$$Y = \sqrt{2}Q\cos\left(\frac{p_Q}{2Q}\right), \tag{28}$$

$$\pi_Y = \sqrt{2}Q\sin\left(\frac{p_Q}{2Q}\right). \tag{29}$$

From the above two formulas, we can easily get

$$Q^2 = \frac{1}{2}(\pi_Y^2 + Y^2). \tag{30}$$

Substituting (30) into (27), we have

$$A = 4\pi(\pi_X^2 + X^2) + 2\pi(\pi_Y^2 + Y^2). \tag{31}$$

After this quantization procedure, the X, Y, π_X and π_Y are considered as coordinate operators and momentum operators, $\widehat{\pi_X} = -i\frac{d}{dX}$ and $\widehat{\pi_Y} = -i\frac{d}{dY}$. And they satisfy the commutation relations $[\widehat{X}, \widehat{\pi_X}] = [\widehat{Y}, \widehat{\pi_Y}] = i$. So the operator corresponding to (31) can be written as

$$\widehat{A} = 4\pi(\widehat{\pi_X}^2 + \widehat{X}^2) + 2\pi(\widehat{\pi_Y}^2 + \widehat{Y}^2). \tag{32}$$

Now we set $X = 2\sqrt{2\pi}\eta$ and $Y = 2\sqrt{\pi}\xi$. Following the standard procedures of quantization, (32) can be reduced to

$$\widehat{A} = \frac{1}{2}(\widehat{\pi_\eta}^2 + \omega_1^2\widehat{\eta}^2 + \widehat{\pi_\xi}^2 + \omega_2^2\widehat{\xi}^2) \tag{33}$$

which is similar to the Hamiltonian of a two-mode harmonic oscillator. Here $\omega_1 = 8\pi$, $\omega_2 = 4\pi$. It can be easily proved that $[\widehat{\eta}, \widehat{\pi_\eta}] = [\widehat{\xi}, \widehat{\pi_\xi}] = i$, namely, they still satisfy the commutation relations.

3 Quantum Area Spectrum of Reissner-Nordström Black Hole

In order to obtain the quantum area spectrum and the horizon area's wave function of Reissner-Nordström black hole, it is convenient to change the coordinate representation to the particle number representation. In the particle number representation, the ladder operators can be defined as

$$\widehat{a}^\dagger = \frac{1}{\sqrt{2\omega_1}}(\omega_1\widehat{\eta} - i\widehat{\pi_\eta}), \tag{34}$$

$$\widehat{a} = \frac{1}{\sqrt{2\omega_1}}(\omega_1\widehat{\eta} + i\widehat{\pi_\eta}), \tag{35}$$

$$\widehat{b}^\dagger = \frac{1}{\sqrt{2\omega_2}}(\omega_2\widehat{\xi} - i\widehat{\pi_\xi}), \tag{36}$$

$$\widehat{b} = \frac{1}{\sqrt{2\omega_2}}(\omega_2\widehat{\xi} + i\widehat{\pi_\xi}). \tag{37}$$

With the above four formulas, (33) can be reduced to

$$\widehat{A}=\omega_1\left(\widehat{a}^{\dagger}\widehat{a}+\frac{1}{2}\right)+\omega_2\left(\widehat{b}^{\dagger}\widehat{b}+\frac{1}{2}\right). \tag{38}$$

And these ladder operators satisfy the following relation, $[\widehat{a},\widehat{a}^{\dagger}]=[\widehat{b},\widehat{b}^{\dagger}]=1$, $[\widehat{a},\widehat{b}]=[\widehat{a}^{\dagger},\widehat{b}^{\dagger}]=[\widehat{a}^{\dagger},\widehat{b}]=[\widehat{a},\widehat{b}^{\dagger}]=0$. Now it is clear that Reissner-Nordström black hole can be reduced to a two-mode Bose harmonic oscillator model by the suitable transformations. And later, we will show that the horizon area operator of Reissner-Nordström black hole just corresponds to this two-mode Bose harmonic oscillator's Hamilton. Firstly, the number operators $\widehat{N_1}=\widehat{a}^{\dagger}\widehat{a}$, $\widehat{N_2}=\widehat{b}^{\dagger}\widehat{b}$ are defined, and their eigenstate is $|n_1,n_2\rangle$, so we have

$$\begin{aligned}\widehat{A}|n_1,n_2\rangle &= \left[\omega_1\left(\widehat{a}^{\dagger}\widehat{a}+\frac{1}{2}\right)+\omega_2\left(\widehat{b}^{\dagger}\widehat{b}+\frac{1}{2}\right)\right]|n_1,n_2\rangle \\ &= \left[\omega_1\left(n_1+\frac{1}{2}\right)+\omega_2\left(n_2+\frac{1}{2}\right)\right]|n_1,n_2\rangle \quad n_1,n_2=0,1,2,\ldots. \end{aligned} \tag{39}$$

Therefore, the eigenvalues of the area operator of Reissner-Nordström black hole can be obtained

$$A_{n_1,n_2}=8\pi\left(n_1+\frac{n_2}{2}\right)+A_0 \quad n_1,n_2=0,1,2,\ldots \tag{40}$$

where $A_0=6\pi$ is the area of the ground state of the black hole. In the international system of units, when n_1 and n_2 are larger, the area spectrum of Reissner-Nordström black hole can be expressed as

$$A_{n_1,n_2}\approx 8\pi\left(n_1+\frac{n_2}{2}\right)l_{pl}^2=4\pi n l_{pl}^2. \tag{41}$$

Comparing with (1), obviously, $\gamma=4\pi$ and $n=2n_1+n_2$ is integer. So this result confirms Bekenstein's hypothesis. Considering the relation between the entropy and horizon area of black holes, we can also get the quantum entropy of the scalar field of Reissner-Nordström black hole

$$S_{n_1,n_2}=\frac{\kappa_B}{4l_{pl}^2}A_{n_1,n_2}\approx\frac{2\pi\kappa_B c^3}{\hbar G}\left(n_1+\frac{n_2}{2}\right) \quad n_1,n_2=0,1,2,\ldots \tag{42}$$

where κ_B is the Boltzmann's constant.

4 Wave Function of Reissner-Nordström Black Hole

Using the particle number operators, the following formula can be obtained

$$\langle n_1,n_2|\widehat{N_1}+\widehat{N_2}|n_1,n_2\rangle=n_1+n_2\geq 0. \tag{43}$$

The state $|0,0\rangle$ is the ground state of the states $|n_1,n_2\rangle$. So there must be $(\widehat{a}+\widehat{b})|0,0\rangle=0$. It is easy to prove that

$$|n_1,n_2\rangle=\left[\frac{(\widehat{a}^{\dagger})^{n_1}}{\sqrt{n_1!}}+\frac{(\widehat{b}^{\dagger})^{n_2}}{\sqrt{n_2!}}\right]|0,0\rangle. \tag{44}$$

The space spanned by states $|n_1, n_2\rangle$ is complete, $\sum_{n_1=n_2=0}^{\infty} |n_1, n_2\rangle\langle n_1, n_2| = 1$. And more we have

$$\widehat{a}|n_1, n_2\rangle = \sqrt{n_1}|n_1 - 1, n_2\rangle, \tag{45}$$

$$\widehat{a}^\dagger|n_1, n_2\rangle = \sqrt{n_1 + 1}|n_1 + 1, n_2\rangle, \tag{46}$$

$$\widehat{b}|n_1, n_2\rangle = \sqrt{n_2}|n_1, n_2 - 1\rangle, \tag{47}$$

$$\widehat{b}^\dagger|n_1, n_2\rangle = \sqrt{n_2 + 1}|n_1, n_2 + 1\rangle. \tag{48}$$

The wave function $\langle\eta, \xi|0, 0\rangle$ can be derived by the following formula

$$\begin{aligned} 0 = \langle\eta, \xi|\widehat{a} + \widehat{b}|0, 0\rangle &= \langle\eta, \xi|\left[\frac{1}{\sqrt{2\omega_1}}(\omega_1\widehat{\eta} + i\widehat{\pi}_\eta) + \frac{1}{\sqrt{2\omega_2}}(\omega_2\widehat{\xi} + i\widehat{\pi}_\xi)\right]|0, 0\rangle \\ &= \left[\frac{1}{\sqrt{2\omega_1}}\left(\omega_1\eta + \frac{d}{d\eta}\right) + \frac{1}{\sqrt{2\omega_2}}\left(\omega_2\xi + \frac{d}{d\xi}\right)\right]\langle\eta, \xi|0, 0\rangle. \end{aligned} \tag{49}$$

We can easily get

$$\langle\eta, \xi|0, 0\rangle = C\exp\left(-\frac{\omega_1}{2}\eta^2 - \frac{\omega_2}{2}\xi^2\right) \tag{50}$$

where the number C is a normalization factor, it can be determined by the following formula

$$\begin{aligned} 1 = \langle 0, 0|0, 0\rangle &= \int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\xi\, \langle 0, 0|\eta, \xi\rangle\langle\eta, \xi|0, 0\rangle\rangle \\ &= |C|^2 \int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\xi\, \exp(-\omega_1\eta^2 - \omega_2\xi^2) = |C|^2\frac{\pi}{\sqrt{\omega_1\omega 2}}. \end{aligned} \tag{51}$$

So the normalized factor reads $|C| = (\frac{\sqrt{\omega_1\omega_2}}{\pi})^{\frac{1}{2}}$. Then, the expression of $\langle\eta, \xi|0, 0\rangle$ is

$$\langle\eta, \xi|0, 0\rangle = \left(\frac{\sqrt{\omega_1\omega_2}}{\pi}\right)^{\frac{1}{2}}\exp\left(-\frac{\omega_1}{2}\eta^2 - \frac{\omega_2}{2}\xi^2\right). \tag{52}$$

The normalized eigen wave function of area operator of Reissner-Nordström black hole horizon can be derived with the above relation

$$\begin{aligned} \psi_{n_1,n_2}(\eta, \xi) &= \langle\eta, \xi|n_1, n_2\rangle \\ &= \int_{-\infty}^{+\infty} d\eta' \int_{-\infty}^{+\infty} d\xi' \langle\eta, \xi|\left[\frac{(\widehat{a}^\dagger)^{n_1}}{\sqrt{n_1!}} + \frac{(\widehat{b}^\dagger)^{n_2}}{\sqrt{n_2!}}\right]|\eta', \xi'\rangle\langle\eta', \xi'|0, 0\rangle \\ &= \frac{\sqrt{\alpha_1\alpha_2}}{\sqrt{2^{n_1+n_2}n_1!n_2!\pi}}\left(\alpha_1\eta - \frac{d}{d(\alpha_1\eta)}\right)^{n_1}\left(\alpha_2\xi - \frac{d}{d(\alpha_2\xi)}\right)^{n_2} \\ &\quad \times \exp\left(-\frac{\alpha_1^2\eta^2}{2} - \frac{\alpha_2^2\xi^2}{2}\right) \end{aligned} \tag{53}$$

where $\alpha_1 = \sqrt{\omega_1} = 2\sqrt{2\pi}$, $\alpha_2 = \sqrt{\omega_2} = 2\sqrt{2\pi}$. With the Hermitian polynomial, (48) can be expressed as

$$\Psi_{n_1,n_2}(\eta,\xi) = \frac{\sqrt{\alpha_1\alpha_2}}{\sqrt{2^{n_1+n_2}n_1!n_2!\pi}} e^{-\frac{\alpha_1^2\eta^2+\alpha_2^2\xi^2}{2}} H_{n1}(\alpha_1\eta)H_{n_2}(\alpha_2\xi) \tag{54}$$

where H_n is Hermitian polynomial. The Hermitian polynomials have the following expressions

$$H_{n_1}(\alpha_1\eta) = e^{\frac{\alpha_1^2\eta^2}{2}}\left(\alpha_1\eta - \frac{d}{d(\alpha_1\eta)}\right)^{n_1} e^{-\frac{\alpha_1^2\eta^2}{2}}, \tag{55}$$

$$H_{n_2}(\alpha_2\xi) = e^{\frac{\alpha_2^2\xi^2}{2}}\left(\alpha_2\xi - \frac{d}{d(\alpha_2\xi)}\right)^{n_2} e^{-\frac{\alpha_2^2\xi^2}{2}}. \tag{56}$$

It can be proved that the two equations equal to

$$H_{n_1}(\alpha_1\eta) = e^{\alpha_1^2\eta^2}\left(-\frac{d}{d(\alpha_1\eta)}\right)^{n_1} e^{-\alpha_1^2\eta^2}, \tag{57}$$

$$H_{n_2}(\alpha_2\xi) = e^{\alpha_2^2\xi^2}\left(-\frac{d}{d(\alpha_2\xi)}\right)^{n_2} e^{-\alpha_2^2\xi^2}. \tag{58}$$

Therefore, the normalized eigen wave function of the area operator of Reissner-Nordström black hole can also be expressed as

$$\begin{aligned}\psi_{n_1,n_2}(\eta,\xi) = {}& \frac{\sqrt{\alpha_1\alpha_2}}{\sqrt{2^{n_1+n_2}n_1!n_2!\pi}}(-1)^{n_1+n_2} e^{\frac{\alpha_1^2\eta^2+\alpha_2^2\xi^2}{2}} \\ & \times \frac{d^{n_1}}{d(\alpha_1\eta)^{n_1}}\frac{d^{n_2}}{d(\alpha_2\xi)^{n_2}} e^{-(\alpha_1^2\eta^2+\alpha_2^2\xi^2)}.\end{aligned} \tag{59}$$

5 Conclusion

To summarize, we have looked at the quantum gravity in Reissner-Nordström black hole. To encorporate the thermodynamic information, we assume periodicity of the momentum conjugate to the black hole mass. It is particular to stress that after a couple of canonical coordinate transformations, Reissner-Nordström black hole is reduced to an ideal model—two-mode Bose harmonic oscillator model. Adopting this two-mode Bose harmonic oscillator model, the quantum area spectrum, the quantum entropy and wave function are accurately derived. The discrete spectrum of black hole implies that black hole can only emit and absorb quanta which is not arbitrary but quantized in terms of n_1 and n_2. The wave function helps us to calculate fluctuations of some other mechanical quantity. We believe that our results reveal some intriguing features of the quantum mechanics of black holes, which is significant for studying the space-time structure and the quantum properties of the black hole.

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